Problem 52

Consider the physical quantities m, s, v, a, and t with dimensions $[m] = \text{M}$, $[s] = \text{L}$, $[v] = \text{LT}^{-1}$, $[a] = \text{LT}^{-2}$, and $[t] = \text{T}$. Assuming each of the following equations is dimensionally consistent, find the dimension of the quantity on the left-hand side of the equation: (a) $F = ma$; (b) $K = 0.5mv^2$; (c) $p = mv$; (d) $W = mas$; (e) $L = mvr$.

Solution

Part (a)

Check the units of both sides.

$$\begin{aligned}[F] &= [ma] \\ &= [m][a] \\ &= \text{M} \cdot \text{LT}^{-2} \\ &= \text{MLT}^{-2}\end{aligned}$$

Part (b)

Check the units of both sides.

$$\begin{aligned}[K] &= [0.5mv^2] \\ &= [0.5][m][v^2] \\ &= [0.5][m][v]^2 \\ &= 1 \cdot \text{M} \cdot (\text{LT}^{-1})^2 \\ &= \text{ML}^2\text{T}^{-2}\end{aligned}$$

Part (c)

Check the units of both sides.

$$\begin{aligned}[p] &= [mv] \\ &= [m][v] \\ &= \text{M} \cdot \text{LT}^{-1} \\ &= \text{MLT}^{-1}\end{aligned}$$

Part (d)

Check the units of both sides.

$$\begin{aligned}[W] &= [mas] \\ &= [m][a][s] \\ &= \mathrm{M} \cdot \mathrm{LT}^{-2} \cdot \mathrm{L} \\ &= \mathrm{ML}^2\mathrm{T}^{-2}\end{aligned}$$

Part (e)

Check the units of both sides.

$$\begin{aligned}[L] &= [mvr] \\ &= [m][v][r] \\ &= \mathrm{M} \cdot \mathrm{LT}^{-1} \cdot \mathrm{L} \\ &= \mathrm{ML}^2\mathrm{T}^{-1}\end{aligned}$$